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Supersymmetry breaking, fermion masses and a small extra dimension

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Abstract: We present a supersymmetric model in which the observed fermion masses and mixings are generated by localizing the three generations of matter and the two Higgs fields at different locations in a compact extra dimension. Supersymmetry is broken by the shining method and the breaking is communicated to standard model fields via gaugino mediation. Quark masses, CKM-mixing angles and the μ term are generated with all dimensionless couplings of $\mathcal{O}(1)$. All dimensionful parameters are of order the five-dimensional Planck scale except for the size of the extra dimension which is of order the GUT scale. The superpartner spectrum is highly predictive and is found to have a neutralino LSP over a wide range of parameter space. The resulting phenomenology and interesting extensions of the model are briefly discussed.

Keywords: Supersymmetry Breaking, Quark Masses and SM Parameters, Beyond Standard Model, Supersymmetric Standard Model.

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1. Introduction

Particle physics is littered with energy scales. The known particles have masses which are spread over numerous orders of magnitude. Most hadron masses hover near the scale of presumed quark confinement. However, the masses of the eight light mesons or pseudo-Goldstone bosons, may be parameterized by explicit chiral symmetry breaking in the fundamental theory. In QCD, this breaking for the most part is due to small quark masses.

In the standard model (SM), non-zero quark and charged lepton masses require the electroweak symmetry to be broken. This breaking (EWSB) is accomplished by allowing a scalar field (Higgs boson) to have a vacuum expectation value (VEV) thus giving masses to the W and Z bosons, the gauge fields of the electroweak interaction (minus the photon). One might expect the masses of the charged fermions to be of similar magnitude as the scale of electroweak symmetry breaking. However, instead the masses extend more than five orders of magnitude below the weak scale. One (substantially explored) explanation for this large hierarchy of masses is the existence of some symmetry broken at high energies producing masses suppressed relative to the weak scale.

A more serious hierarchy problem in the SM is the instability of the Higgs mass, and thus the electroweak scale, with respect to quantum corrections. A theory with supersymmetry (SUSY) softly broken at the weak scale, such as the minimal supersymmetric standard model (MSSM), can stabilize the Higgs mass because in such theories all quadratically divergent quantum corrections vanish. However, generically these theories contain contributions to flavor changing neutral currents (FCNC) and CP violation which far exceed current experimental bounds [1]. Again, symmetries could restrict flavor violations among supersymmetry breaking terms. For instance flavor symmetries which also restrict the soft terms [2], gauge symmetries which mediate supersymmetry breaking [3], or some combination which results in partially aligned heavy first two generations [4, 5].

Recently, Randall and Sundrum [6] have suggested a new way to explain small couplings without appealing to symmetries. They were able to forbid generally flavor violating non-renormalizable operators that mix MSSM fields with fields in a supersymmetry breaking sector by spatially separating the two sectors in a small extra dimension. All soft terms appear due to contributions coming from the superconformal anomaly [6, 7]. While the soft terms are sufficiently flavor diagonal in the minimal scenario, the sleptons are tachyonic, and thus break the electromagnetic symmetry. Various model-building scenarios have appeared in the literature which attempt to fix this problem [8].

Arkani-Hamed and Schmaltz have since shown that localizing fields in extra dimensions at distances of order unity with respect to the fundamental scale can easily produce exponentially small Yukawa couplings and at the same time suppress proton decay due to the small overlap the their wave functions [9]. This mechanism is especially useful in models in which large compact extra dimensions solve the hierarchy problem by bringing the fundamental Planck scale down to just above the weak scale [10]. In general, this method presents an interesting alternative to the usual spontaneous flavor symmetry breaking scenarios [12].

In this note, we present a model for fermion masses where Yukawa couplings are suppressed due to the localization of fields in an extra spatial dimension. Our model differs significantly from the Arkani-Hamed/Schmaltz model in three ways:[1]

[1]Dvali and Shifman [16] also considered localizing complete generations and Higgs fields in an extra dimension and suggested a way supersymmetry breaking could be included via non-BPS

(i) our extra dimension is small (tens of Planck lengths) and supersymmetry solves the hierarchy problem, thus avoiding bounds from flavor-violating neutral current interactions induced by the relatively light Kaluza-Klein (KK) excitations of the gauge bosons [11].

(ii) Yukawa couplings are small due to the position of each generation relative to the (localized) Higgs fields and not due to the different splittings of left- and right-handed fermions [13], and

(iii) our gauge fields fill the entire space in the extra dimension thus relieving us of the difficult task of localizing gauge fields in field theory [14, 15].

Because our gauge fields live in the bulk, localized supersymmetry breaking produces the conditions necessary for recently proposed [18, 19] gaugino-mediated supersymmetry breaking ($\tilde{g}$MSB), arguably the simplest way to mediate supersymmetry breaking while avoiding all phenomenological flavor constraints. In addition, by using a particular variation of the "shining" mechanism of Arkani-Hamed, et al. [20], we localize supersymmetry breaking close to the Higgs fields making it possible to both generate a μ term and insure that the right-handed stau is not the lightest supersymmetric particle (LSP).

Our fields are localized by generation and thus are consistent with a supersymmetric SU(5) grand unified theory (GUT). This has implications for charged lepton masses and thus we shall assume that GUT-breaking is responsible for producing the correct leptonic spectrum [21]. A small extra dimension opens new possibilities with regards to GUT models, but we leave this and the leptons for future work.

The paper is organized as follows. Section 2 describes all of the elements necessary for a successful flavor model in this context. We find the model to be quite constrained and predictive. Section 3 describes the incorporation of supersymmetry and supersymmetry breaking, including a brief review of $\tilde{g}$MSB and some attractive modifications with respect to the μ term. Section 4 describes the phenomenology of the model with respect to the quarks and CKM matrix as well as the supersymmetric spectrum. Section 5 discusses possible future enhancements of this model and appendix A describes the localization of an $N = 1$ chiral multiplet .

2. Flavor from extra-dimensional overlaps

In this section we show how, with a small extra dimension, to generate small fermion masses from $\mathcal{O}(1)$ Yukawa couplings. The fermions are localized with respect to the extra dimension and their separations from each other and the Higgs fields are

brane configurations. Also, Gherghetta and Pomarol have recently suggested that small Yukawa couplings could arise from localizing fermions at different positions in a slice of anti-de Sitter space and discuss variations of supersymmetry breaking in such scenarios [17].

also $\mathcal{O}(1)$ (in Planck units). Supersymmetry does not play a role in this discussion, other than to motivate the existence of two Higgs doublets, and thus the results of this section may be applied generically to any supersymmetric theory. The details concerning the localization of a zero mode of a chiral superfield with a gaussian profile are presented in appendix A. In this work we will only consider the quark masses and mixings. The lepton masses may be obtained from a straight-forward generalization of these results.

2.1 Gaussian localized MSSM fields

We take as our starting point a five-dimensional (5 d) GUT. The fifth dimension is small, with associated mass scale M_c of order 1/100 times the four-dimensional (4 d) Planck scale. Thus, at energies below M_c, there is a 4 d effective theory description of the resulting dynamics. The fermion masses arise from 5 d superpotential terms, written conveniently in the language of 4 d $N = 1$ supersymmetry [20] as,

$$\int dy \int d^2\theta \sum_{i,j=1}^{3} \left\{ \frac{Y^u_{ij}}{\sqrt{M_*}}\, H_u\, Q_i\, U^c_j + \frac{Y^d_{ij}}{\sqrt{M_*}}\, H_d\, Q_i\, D^c_j \right\} + \text{h.c.}\,, \tag{2.1}$$

where H_u and H_d are the chiral superfields containing the up- and down-type Higgses, Q is the quark SU(2) doublet, and U^c and D^c are the up- and down-type quark SU(2) singlets. The 5 d Planck scale M_* is related to the 4 d Planck scale by $M_* = (M_p^2/L)^{1/3}$, y is the coordinate parameterizing the compact dimension and powers of M_* have been inserted such that Y_{ij} are dimensionless. This superpotential violates the $N = 1$ supersymmetry in 5 dimensions but is invariant under half of the supersymmetry transformations which correspond to $N = 1$ supersymmetry in 4 dimensions. One could imagine explicit breaking in a microscopic theory where at least some of the fields (either quarks or Higgses) in (2.1) live on "3-branes" which break translation symmetry in the fifth coordinate along with half of the supersymmetries.[2] We also assume it is possible to localize only a single left-handed zero-mode, without also introducing a right-handed one. We will not explore this subtle but difficult issue and assume it can be accomplished. Localizing a field using the method outlined in appendix A also explicitly breaks half the supersymmetries and again we take this as a requirement of the form of explicit "$N = 2$" supersymmetry breaking and leave this issue for future work.[3] In addition, we assume the chiral superfield component of the 5 d gauge multiplet can be given an explicit mass on a 3-brane, or removed

[2]This assumption could lead to fields with a simple exponential fall-off rather than the gaussian profile described below. We have checked to see that successful models could be produced with simple exponentials and found similar results to those described here. Thus, for simplicity, we assume all fields have gaussian wave functions.

[3]We thank Martin Schmaltz for discussions on these issues.

from the low energy theory in some way (for example, see [25]). Here we take a bottom-up approach in constructing the model, allowing successful phenomenology to motivate the high-energy theory.

The Higgs and fermion fields are localized in the fifth dimension with gaussian profiles. We will assume that the zero modes of the quarks of a given family are fixed to same location in y, and that the gaussian profiles of the zero modes for all three families have a common width $2/\zeta^2$ (which we take to be about the Planck scale). This is consistent with GUT scenarios and would arise if all matter was localized by the same mass function (or VEV) whose slope does not significantly vary over the positions of the localized fields. The localization of each family around a *different* point in y can be accomplished by giving each family's hypermultiplets different constant mass terms in addition to the single mass profile which results in the zero modes [9]. Thus, provided the extent of the extra dimension is large enough that the deviations from this gaussian profile are small, the zero mode profile for fermion j is given by,

$$\psi_j^0(y) = \left(\frac{2\,\zeta_j^2}{\pi} \right)^{1/4} e^{-\zeta_j^2 (y - l_j)^2} . \tag{2.2}$$

The Higgs superfields are also localized in y, and we further allow them to have different widths from the fermions and from each other. For now we will assume that all of the Yukawa interactions Y_{ij} are exactly 1. We will see below that a phase (required by CP violation) will also be important in obtaining the correct mixing angles.

The resulting low energy effective theory has exponentially suppressed Yukawa interactions that result from the overlap of any two fermion wave function with a Higgs wave function. We define our coordinate system such that the up-type Higgs is at $y = 0$. We measure distances in y in units of $1/\zeta$, and thus the model is completely specified by the locations of the three families and down-type Higgs, l_1, l_2, l_3 and l_h, and the relative widths of the Higgses, $r_u = \zeta_{H_u}/\zeta$ and $r_d = \zeta_{H_d}/\zeta$. In terms of these quantities, the resulting 4 d Yukawa interactions for up-type quarks with H_u is,

$$y_{ij}^u = \sqrt{\frac{r_u}{2 + r_u^2}} \left(\frac{2^{3/4}}{\pi^{1/4}} \right) \exp\left[-\frac{1}{2 + r_u^2} \Big(\left(1 + r_u^2\right) \left(l_i^2 + l_j^2\right) - 2\, l_i\, l_j \Big) \right] , \tag{2.3}$$

where Y_{ij}^u has been taken to be one, and ζ to be M_*. There are also interactions between the down-type quarks and H_d of the same form, but with $r_u \to r_d$ and $l_i \to (l_i - l_h)$. When the up- and down-type Higgs scalars obtain VEV's v_u and v_d, these interactions will provide Dirac mass matrices for the quarks. These matrices may be diagonalized by separately rotating the right- and left-chiral fields, resulting in six-real quark masses, and the three-mixing angles and one complex phase that make up the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Thus, our first task is to see if there exists a suitable arrangement of parameters to match the low energy data.

2.2 Fermion masses and the CKM matrix

We will now outline a method to determine our flavor parameters, l_1, l_2, l_3, l_h, r_u and r_d, in order to fit the low energy data. We start with the $\overline{\text{MS}}$ quark masses and (90% C.L.) three generation CKM matrix [22],

$$\begin{aligned} m_u(2\,\text{GeV}) &= 1.5-5\,\text{MeV}\,, \qquad & m_c(m_c) &= 1.1-1.4\,\text{GeV}, \\ m_d(2\,\text{GeV}) &= 3-9\,\text{MeV}\,, \qquad & m_b(m_b) &= 4.1-4.4\,\text{GeV}, \\ m_s(2\,\text{GeV}) &= 60-170\,\text{MeV}\,, \qquad & m_t(m_t) &= 161-171\,\text{GeV}\,, \end{aligned} \tag{2.4}$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.9742-0.9757 & 0.219-0.226 & 0.002-0.005 \\ 0.219-0.225 & 0.9734-0.9749 & 0.037-0.043 \\ 0.004-0.014 & 0.035-0.043 & 0.9990-0.9993 \end{pmatrix}.$$

We will work at the top mass scale[4] (using the three-loop QCD and one-loop QED renormalization group scaling factors of [23] to find the light quark masses at m_t).

The measured Z boson mass requires $v = \sqrt{v_u^2 + v_d^2} = 246\,\text{GeV}$, while the ratio $\tan\beta = v_u/v_d$ remains unfixed by EWSB. We will treat $\tan\beta$ as a prediction of our model, and characterize the allowed range of $\tan\beta$ by what results from the set of model parameters which accurately predict the quark masses and mixings. The correct top mass in eq. (2.3) is obtained by fixing the magnitude of l_3 once r_u is chosen. (One can think that $l_3(r_u)$ is fixed by m_t to be a function of a chosen r_u.) In the spirit of our work, we want to invoke numbers of $\mathcal{O}(1)$, and thus we allow the widths to vary at most between 1/2 and 2. For $v_u \sim v$ and $r_u \sim 1.5$, this requires $l_3 \sim 0.3$, though there is some freedom to vary l_3 and r_u together. Requiring that the ratio m_t/m_c come out correctly requires us to choose an appropriate $l_2(l_3, r_u) = l_2(r_u)$. For the particular numbers discussed above the working choice is $l_2 \sim 2.3$.

Turning to the down-type quarks, the ratio m_b/m_s, combined with the already "chosen" value of r_u and the "determined" values of l_3 and l_2 fixes a combination of r_d and l_h. The magnitude of m_b determines $\tan\beta$ in terms of the above parameters. We may now choose l_1 in order to arrive at the correct CKM element V_{us}. As it stands, we have chosen two parameters (r_u and r_d), and used 4 pieces of experimental data (m_t, m_t/m_c, m_b/m_s and V_{us}) to determine the remaining parameters l_1, l_2, l_3 and l_h. It remains to determine whether we can accommodate the remaining experimental data: V_{ub}, V_{cb} and the first family quark masses by varying r_d and r_u independently over the "reasonable" range of 1/2 to 2. (One can think that two of the remaining observables fix r_d and r_u, and the last two are predictions of the model.)

[4]The 5 d Yukawa interactions in section 2.1 are actually those at M_c. However, as the quark masses evolve together between m_t and M_c, this can simply be corrected by rescaling all of the 5 d Yukawa interactions.

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As it turns out, the answer is no. While we easily realize the correct order of magnitude for the remaining predictions, we cannot quite reach the experimental values. V_{cb} is always at least a factor of two or so smaller than its measured value. This is not really a serious failing; we have set out to construct a model in which all of the 5 d Yukawa interactions were $\mathcal{O}(1)$, but we have tried to realize the model by taking the couplings to be strictly one. This will never produce the CP violation observed in nature [22], and thus it is obviously too naive. In fact, taking a simple ansatz that the 5 d Yukawa interactions have the phase structure,

$$Y^u = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad Y^d = \begin{pmatrix} 1 & i & 1 \\ i & 1 & i \\ 1 & i & 1 \end{pmatrix}, \tag{2.5}$$

we find that it is quite easy to fit the low energy data for a range of l_h and r_d. This particular choice produces a CP-violating phase of $\mathcal{O}(1)$, in accordance with measured CP violation in the Kaon system. We have verified that the addition of phases to the 5 d Yukawa interactions greatly extends the workable range of l_1, l_2, l_3, l_h, r_u and r_d. Thus, in our context the phases are not only critical for CP violation, but also to get the right magnitude for the CKM elements. We stress that the ansatz (2.5) is only one of a wide range of workable solutions, which is a general indication of the robustness of the result.

As a particular example of a working parameter set, consider the model defined by $r_u = 1.5$, $r_d = 0.67$, $l_1 = 3.05$, $l_2 = 2.29$, $l_3 = 0.36$ and $l_h = -1.7$. The gaussian profiles for the zero modes are shown graphically in figure 1. The physical origin of this solution to the flavor problem is evident from the positions and widths of the profiles. The third family sits close to the up-type Higgs to produce the large value of m_t. The first and second generations are rather close together compared to the third generation, in order to correctly generate $V_{us} > V_{cb} > V_{ub}$. The different widths and locations of the Higgs profiles account for the fact that $m_t > m_b$ and $m_c > m_s$, whereas $m_d > m_u$. The wider down-type Higgs profile allows it more overlap with the first generation relative to the narrow up-type Higgs profile, which is very small at the distant location of the first family. This results in low energy predictions,

$$\begin{aligned} m_u(2\,\mathrm{GeV}) &= 3.6\,\mathrm{MeV}\,, \qquad & m_c(m_c) &= 1.3\,\mathrm{GeV}\,, \\ m_d(2\,\mathrm{GeV}) &= 8.8\,\mathrm{MeV}\,, \qquad & m_b(m_b) &= 4.3\,\mathrm{GeV}\,, \\ m_s(2\,\mathrm{GeV}) &= 75.0\,\mathrm{MeV}\,, \qquad & m_t(m_t) &= 166\,\mathrm{GeV}\,, \end{aligned} \tag{2.6}$$

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.9755 & 0.221 & 0.0046 \\ 0.220 & 0.9748 & 0.038 \\ 0.0076 & 0.037 & 0.9992 \end{pmatrix}.$$

Comparison of eqs. (2.4) and (2.6) indicates that we have succeeded admirably in satisfying the low energy flavor measurements. The CKM elements fit comfortably

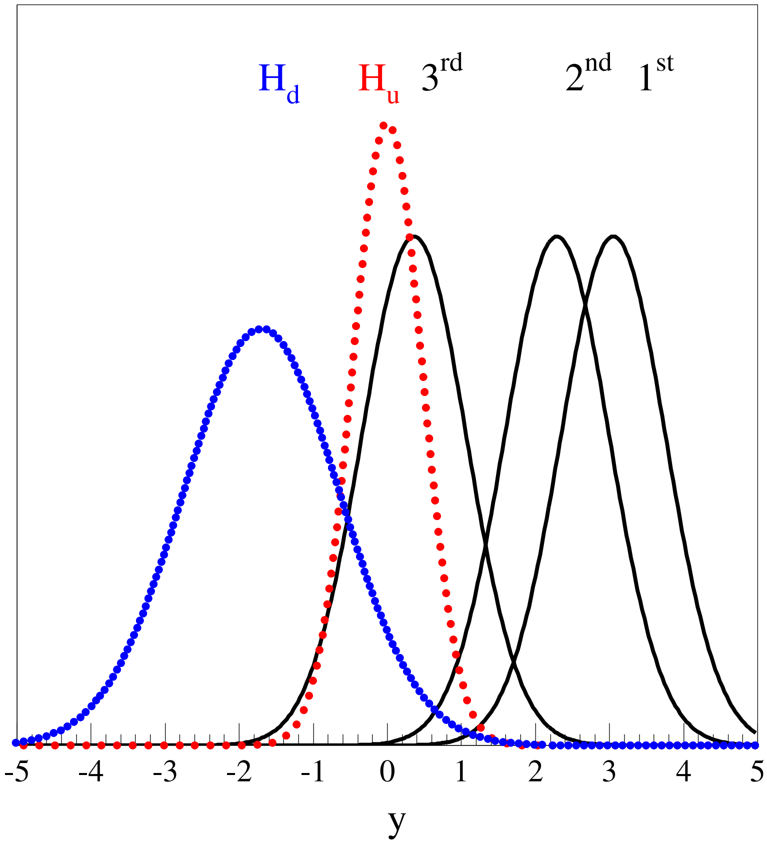

Figure 1: Zero mode profiles for the particular parameter set with $l_1 = 3.05$, $l_2 = 2.29$, $l_3 = 0.36$, $l_h = -1.7$, $r_u = 1.5$ and $r_d = 0.67$.

into their allowed ranges, and the quark masses are all reasonable. Furthermore, the quantities $(m_u + m_d)/2 = 6\,\text{MeV}$, and $m_u/m_d = 0.4$ are more rigorously defined from Chiral Perturbation Theory (CPT), and fall into the acceptable ranges [22].

This particular model has $\tan\beta = 13$. In fact there is some remaining freedom to move around in the acceptable ranges for the quark masses and mixings, provided all of the parameters are suitably adjusted together. This can be characterized as l_h, which shows the greatest freedom to move consistently with the data. In figure 2 we show a scatter plot of models consistent with low energy data, in the plane of l_h and the resulting $\tan\beta$. As can be seen from the figure, l_h can vary between about -0.5 and -2.4, with $\tan\beta$ running from 45 to about 8 in that range. The cut-off at $l_h = -2.4$ occurs because going lower requires $r_d < 1/2$.

3. Supersymmetry breaking

In this section we introduce supersymmetry breaking into the model described above. Since our model involves a small extra dimension with SM gauge fields in the bulk, it naturally lends itself to the mechanism of gaugino-mediated supersymmetry break-

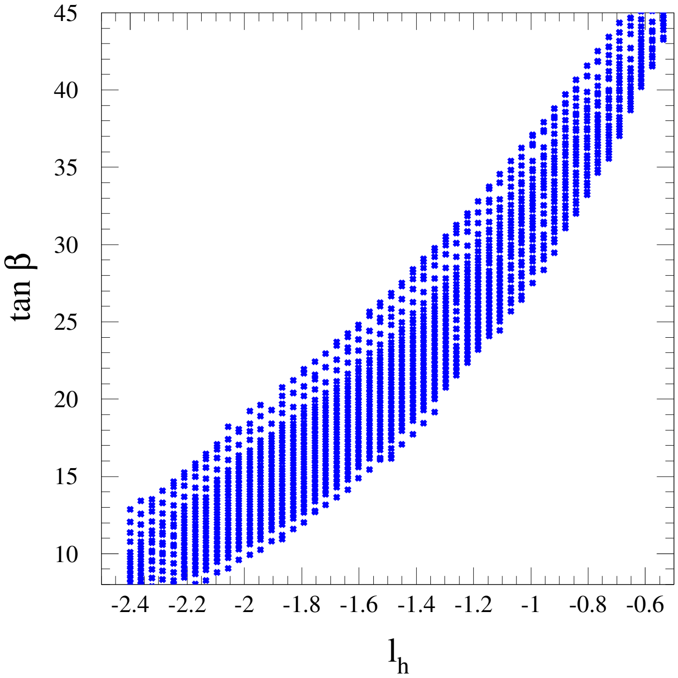

Figure 2: Scatter plot for models with various l_h and the resulting $\tan\beta$.

ing [18, 19]. If we use the shining mechanism to break supersymmetry [20], we can naturally produce a weak-scale μ term still keeping all coefficients of $\mathcal{O}(1)$. We shall see that a general consequence of this μ term solution is a non-zero soft mass for the down-type Higgs at the high scale. This results in a neutralino LSP in a large region of parameter space and thus we avoid bounds on stable charged particles.

3.1 Gaugino mediated supersymmetry breaking

It was realized a number of years ago that a very simple set of boundary conditions on soft terms, namely a non-zero gaugino mass at a high scale, would produce a theory at the weak scale with scalar masses and gaugino masses of the same size, suppressed contributions to FCNC and successful EWSB. Models with these boundary conditions were called "no scale" models [24]. Gaugino mediation [18, 19] gives (so far) the simplest realization of this spectrum from a microscopic theory.

For $\tilde{g}$MSB to work, we need a small ($\sim 10-100$ Planck lengths) extra dimension in which the SM gauge fields and their superpartners propagate. We also require supersymmetry to break on a 4 d hypersurface which is separated in the fifth dimension from the hypersurface(s) on which MSSM matter lives. Thus contributions to scalar masses via Planck-suppressed operators are also exponentially suppressed as Yukawa couplings are in the previous section.

The effective 4 d operator which contributes to gaugino masses is

$$\int d^2\theta \, \frac{S}{M_*(M_*L)} \, W^\alpha W_\alpha \, , \tag{3.1}$$

where L is the size of the extra dimension, W^α is a chiral superfield whose lowest component is the gaugino and S is a gauge singlet. The field S lives in the supersymmetry breaking sector and has a non-zero VEV in its auxiliary component, F_S. As a result, this term produces a gaugino mass at the compactification scale of order $M_{1/2} \sim \frac{F_S}{M_*(M_*L)}$.

The operators which contribute to squared scalar masses are

$$\int d^4\theta \, \frac{S^\dagger S}{M_*^2(M_*L)} \, Q_i^\dagger \, Q_j \, , \tag{3.2}$$

where Q_i are chiral superfields which contain MSSM matter and i, j are flavor indices. The operators with $i \neq j$ would normally make dangerous contributions to FCNC. However, all of the terms in (3.2) are multiplied by exponentially suppressed coefficients due to the spatial separation between the supersymmetry breaking sector and MSSM matter fields, and thus these terms may be ignored provided the supersymmetry breaking sector is sufficiently distant from all MSSM matter. This suppression was not operative in the gaugino mass operators because the gauginos are not localized in y.

The dominant contribution to scalar masses comes from the renormalization group (RG) evolution[5] from the compactification scale $M_c \equiv 1/L$ to the weak scale. The relevant term in the one-loop beta function is

$$\frac{d}{dt} \, m_{\tilde{f}}^2 = - \sum_i \frac{g_i^2}{2\pi^2} \, C_i(r_f) \, M_i^2 \, , \tag{3.3}$$

where $m_{\tilde{f}}^2$ are the squared scalar masses, M_i are the gaugino masses and $C_i(r_f)$ is the quadratic casimir for representation r_f of chiral superfield f in gauge group i. As in gauge mediation, these contributions are only proportional to gauge couplings, and therefor are flavor diagonal and do not contribute to FCNC or CP violation. If we take $M_c \sim M_{\mathrm{GUT}}$, the loop factor suppression is matched by a large logarithm such that scalar masses and gaugino masses are comparable.

There is also a contribution to the scalar masses which dominates at the compactification scale. It can be considered a threshold correction from integrating out the higher KK modes of the gaugino. It comes from the loop contribution depicted in figure 3, where gauginos run in the loop and the operators responsible for gaugino masses are inserted on the propagators at the location of supersymmetry breaking [18, 19]. Contributions like this one were calculated in [18] assuming that the size of the extra dimension and the distance between supersymmetry breaking and MSSM matter are the same (see also [25]). The latter will not be the case in our model and these contributions can be important as we discuss below and in section 4.

[5] Our RG analysis is carried out at two loops with respect to α_S (with one loop thresholds) and at one loop with respect to all other quantities using the beta functions of [26].

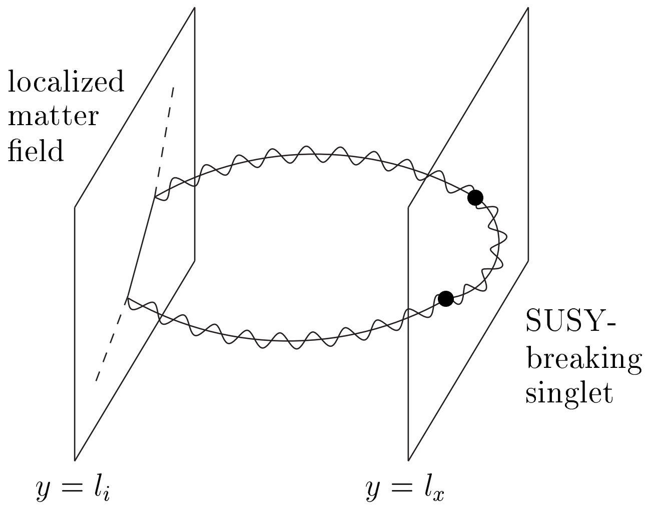

Figure 3: Loop contribution to scalar masses from localized supersymmetry breaking.

One remaining superpartner mass which we have not yet specified is that of the Higgsino. The μ term, which is a superpotential mass term mixing the up- and down-type Higgses (H_u and H_d), should be generated dynamically to explain its weak scale value required by radiative EWSB. By putting the Higgs fields in the bulk [28, 19], the μ term can be produced via the Giudice-Masiero mechanism [29]. It was also pointed out [30] that the shining mechanism [20], which could break supersymmetry on a distant hypersurface, could also be used to produce a μ term on our brane. Both solutions require somewhat small couplings, the former for soft masses in the Higgs sector and the latter for the μ term itself.

In the standard picture of $\tilde{g}$MSB, with $M_c = M_{\rm GUT}$ and Higgs fields localized with MSSM matter, an electrically charged stau is the LSP, a scenario which is disfavored by cosmological considerations. The lightest neutralino can become the LSP if either the Higgs fields are in the bulk and the soft mass of H_d is somewhat larger than the soft mass of H_u [19] or $M_c > M_{\rm GUT}$ by at least an order of magnitude so that the scalar masses run significantly above the GUT scale [30].

Here we offer modified solutions to the μ and LSP problems which naturally fit into our model of flavor. The mechanism allows all couplings to be of $\mathcal{O}(1)$ and produces simple high-scale boundary conditions (though the location of supersymmetry breaking is somewhat fine-tuned). The weak scale spectrum is in general distinguishable from other models of $\tilde{g}$MSB.

3.2 Soft parameters and the μ term

To break supersymmetry in our model, we will use the shining mechanism of Arkani-Hamed, et al. [20]. One advantage of shining is that it does not require the localization of gauge fields, a difficult task in more than four dimensions [14, 15]. Shining is also advantageous as it allows for new solutions to the μ problem [20, 30]. We discuss two variations of the solution of Schmaltz and Skiba in minimal gaugino mediation [30] and show in our context how to remove the requirement for a small coupling. We also show that in this context that a neutralino can be easily be the LSP even if $M_c = M_{\rm GUT}$.

3.2.1 Scenario 1

We introduce two chiral superfields, Φ and Φ^c, to the bulk which are singlets under the SM gauge groups. These fields together are a hypermultiplet of the $N = 1$ supersymmetry in 5 d, which is conserved by this mechanism up to explicit breaking by a source J^c which couples to Φ in the superpotential. This source is localized at $y = l_s$ near the MSSM fields. In the absence of further ingredients, the scalar component of Φ^c will acquire a VEV and supersymmetry remains unbroken. Supersymmetry is broken by introducing another singlet chiral superfield X localized in the bulk far from our MSSM matter, and coupling to Φ^c. Assuming X has a profile $\psi^0_x(y)$ with respect to the compact dimension, the Lagrange density can be expressed,

$$\begin{aligned}\mathcal{L} = &\int d^4\theta \int dy \left[\Phi(y)\,\Phi^\dagger(y) + \Phi^c(y)\,\Phi^{c\dagger}(y) + \left|\psi^0_x(y)\right|^2 X X^\dagger\right] + \\ &+ \int d^2\theta \int dy \Big[\Phi^c(y)\,(\partial_y + m_\phi)\,\Phi(y) + J^c\,\Phi(y)\,\delta\,(y - l_s) + \eta^c \psi^0_x(y)\,X\,\Phi^c\Big] + \\ &+ \text{h.c.}\,,\end{aligned} \tag{3.4}$$

where m_ϕ, J^c and η^c are all dimensionful couplings of order the Planck scale to the appropriate power. We take $\psi^0_x(y)$ normalized such that $\int dy\,|\psi^0_x(y)|^2 = 1$ in order to produce canonically normalized kinetic terms. The F-term equations are

$$|F_\phi(y)| = \Big|\,(-\partial_y + m_\phi)\,\phi^c(y) + J^c\,\delta(y - l_s)\Big|\,, \tag{3.5}$$

$$|F_{\phi^c}(y)| = \Big|\,(\partial_y + m_\phi)\,\phi(y) + \eta^c\,\psi^0_x(y)\,X\Big|\,, \tag{3.6}$$

$$\left|F^0_X\right| = \left|\eta^c \int dy\,\psi^0_x(y)\,\phi^c(y)\right|\,. \tag{3.7}$$

For X localized far from l_s, the potential is minimized with a non-zero VEV for ϕ^c:

$$\langle\phi^c\rangle \simeq -\theta(l_s - y)\,J^c\,e^{-m_\phi(l_s - y)}\,. \tag{3.8}$$

If $\psi^0_x(y)$ is a narrow function localized around the point $l_x < l_s$, then the F-term conditions ($F_i = 0$) cannot be simultaneously met and thus supersymmetry is broken. The field X now plays the role of the hidden sector singlet[6] in eqs. (3.1) and (3.2).

[6] Again the full 5 d $N = 1$ supersymmetry is broken by such couplings. We assume that the theory above M_* accounts for this specific breaking pattern. A simple high-energy description would be to confine X to a brane localized in y, but we shall leave the discussion more general.

As noted below eq. (3.2), provided X is sufficiently distant from the MSSM matter, it will provide negligible soft masses to the sfermions.

A μ term can be generated [30] by adding the following terms to the lagrangian:

$$\int d^2\theta \Big[J\,\Phi^c(y)\,\delta(y-l_x) + \lambda_\mu\,\psi^0_{H_u}(y)\,\psi^0_{H_d}(y)\,H_u\,H_d\,\Phi(y) \Big]\,. \tag{3.9}$$

Using the notation of section 2, including the redefinition $l_i \to l_i/\zeta$, and taking $|l_x| \gg 1$ we find

$$\begin{aligned}
\mu &= \int_{l_x}^{\infty} dy\,\sqrt{\frac{2}{\pi}}\,\lambda_\mu\,J\,\sqrt{r_u r_d}\,\exp\Big[-r_u^2 y^2 - r_d^2\,(y-l_h)^2 - r_\phi\,(y-l_x)\Big] \\
&\simeq \lambda_\mu\,J\,\sqrt{\frac{2 r_u r_d}{r_u^2+r_d^2}}\,\exp\left[r_\phi\,l_x - \frac{r_u^2\,r_d^2\,l_h^2 + r_d^2\,l_h\,r_\phi + (r_\phi/2)^2}{r_u^2+r_d^2}\right] \\
&\simeq M_*\exp\left[r_\phi\,l_x - \frac{r_u^2\,r_d^2\,l_h^2 + r_d^2\,l_h\,r_\phi + (r_\phi/2)^2}{r_u^2+r_d^2}\right],
\end{aligned} \tag{3.10}$$

where $r_\phi = m_\phi/\zeta$. The first term in the exponential dominates and is negative as presumed above. Taking $\psi^0_x(y) = (2\,\zeta_x^2/\pi)^{1/4} e^{-\zeta_x^2 (y-l_x)^2}$ and $r_x \equiv \zeta_x/\zeta$, we get a gaugino mass at the compactification scale of

$$\begin{aligned}
M_{1/2} &= \int_{-\infty}^{l_s} dy\,\left(\frac{2}{\pi}\right)^{1/4} \frac{\eta^c J^c}{\sqrt{\zeta}\,M_*(M_* L)}\,\sqrt{r_x}\,\exp\Big[-r_x^2\,(y-l_x)^2 + r_\phi\,(y-l_s)\Big] \\
&\simeq (2\pi)^{1/4}\frac{\eta^c J^c}{\sqrt{\zeta r_x}\,M_*(M_* L)}\,\exp\left[-r_\phi\,(l_x-l_s) + \left(\frac{r_\phi}{2 r_x}\right)^2\right] \\
&\sim M_c \exp\left[r_\phi\,(l_x-l_s) + \left(\frac{r_\phi}{2 r_x}\right)^2\right].
\end{aligned} \tag{3.11}$$

The last lines of eqs. (3.10) and (3.11) come from taking $\eta^c \sim J^{2/3} \sim (J^c)^{2/3} \sim \zeta_x \sim M_*$. By making $l_h \sim l_s +$ (a few), the μ term and $M_{1/2}$ will be the same size. Again, a small coupling (required in [30]) can be generated by an $\mathcal{O}(1)$ distance (in Planck units).

Even though X is fixed far from the MSSM matter fields, loop contributions of the type in figure 3 can still play an important role in the low-energy spectrum if the distance between X and the matter fields, $\Delta l \equiv (l_i - l_x)$, is a fraction of the total size L of the extra dimension. The contribution from these loops were calculated in [18] for the case of $\Delta l = L$. For $\Delta l \ll L$, the loop integrals are cut off above the mass of $N_{\rm KK} \sim (L/\Delta l)$ Kaluza-Klein modes. The integrals require the 5 d gaugino propagator [18, 31] expressed in position space in the fifth dimension and in momentum space in the 4 large dimensions,

$$P_5(q;x) = \left(\frac{2}{L}\right)\sum_n^{N_{KK}} \frac{e^{i p_n \Delta l}\;(\gamma^\mu\,q_\mu + i\,\gamma_5\,p_n)}{q^2 - p_n^2}\,, \tag{3.12}$$

where $p_n = n\pi/L$ is the (quantized) momentum flowing the compact dimension. The sum can be carried out and results in a simple expression in terms of hyperbolic functions [31]. Inserting this propagator in the Feynman diagram shown in figure 3 the contribution to the scalar masses may be estimated as,

$$m^2 \sim \frac{g_4^2}{16\,\pi^2}\, M_{1/2}^2 \int_1^\infty dq\, \frac{q^2 \cosh^2[q(1-x)]}{\sinh^2[q]\tanh[q]}\,, \tag{3.13}$$

where $x = \Delta l/L$, and we have rescaled q by $1/L$ so that the leading L dependence is absorbed into $M_{1/2}$ and g_4, resulting in the 4 d effective quantities appearing in the equation.

We have inserted the entire set of SU(5) gauginos into the loops, and assumed them all to be massless above the compactification scale. This result should be regarded as an order of magnitude estimate, because once the theory begins to look five dimensional, the gauge coupling will experience strong running, and though these effects appear in the calculation formally at higher order, it may be important to resum them by including the running of the coupling with q. We estimate these effects to be a factor of a few.

These contributions can in principal be larger than one-loop threshold corrections and should be included in our high-scale boundary conditions. Distances of $\Delta l \leq L/4$ could lead to contributions which shift the LSP from a stau to a neutralino. What is interesting about these contributions is that while they are flavor *diagonal* they are not flavor *independent* due to the different locations of the generations. For a wide range in parameter space (Δl vs. L), these contributions are below the bounds on additional FCNC and CP violation [1] and leave a distinct imprint on the spectrum.

3.2.2 Scenario 2

For variety, we could move X from where J is to where J^c is (near the matter fields) and remove J^c altogether. This corresponds to exchanging l_x and l_s above. In this way, ϕ's VEV is responsible for both supersymmetry breaking and the μ term. In addition, the coupling $\eta^c X \Phi^c$ must be exchanged for $\eta\, X \Phi$. In principal this would work while giving a non-negligible contribution to H_d from Planck suppressed operators.

For successful gaugino mediation, gravity-mediated contributions to squark and slepton masses must be small. Thus, this essentially puts a restriction on how close X can be localized to the matter fields. For a squark positioned at l_i, its squared mass m_i^2 receives a contribution of

$$m_i^2 \sim \frac{F_x^2}{M_*^2} \exp\left[-\frac{2r_x^2}{r_x^2+1}\left[(l_i - l_x)^2\right]\right] \tag{3.14}$$

while an off-diagonal mass squared m_{ij}^2 with squarks localized at l_i and l_j receives a contribution

$$\frac{F_x^2}{M_*^2} \exp\left[-\frac{1}{2(r_x^2+1)}\Big[(l_i-l_j)^2+2r_x^2\Big((l_i-l_x)^2+(l_j-l_x)^2\Big)\Big]\right], \tag{3.15}$$

which is sufficiently suppressed for distances of order a few units and r_x of $\mathcal{O}(1)$. Thus, only H_d receives a non-negligible contribution to its soft mass for most of parameter space.

However, the loop contributions described in scenario 1 are too large in this scenario because X is so close to the matter fields. This problem can be remedied by altering the shining mechanism. If instead of the coupling $\eta\, X\Phi$ we add the coupling $\lambda_x\, X\Phi^2$ then we find for F_X:

$$|F_X| = \left|\lambda_x \int dy\, \psi_x^0(y)\, \phi(y)^2\right|, \tag{3.16}$$

thus requiring l_x to have a more intermediate value between l_h and l_s. The μ term in this scenario is the same as in eq. (3.10) if one replaces l_x with l_s, while the universal gaugino mass is

$$\begin{aligned} M_{1/2} &\simeq (2\pi)^{1/4} \frac{\lambda_x J^2}{\sqrt{\zeta_x}\, M_*(M_*L)} \exp\left[-2r_\phi\,(l_x-l_s)+\left(\frac{r_\phi}{r_x}\right)^2\right] \\ &\sim M_c \exp\left[-2r_\phi\,(l_x-l_s)+\left(\frac{r_\phi}{r_x}\right)^2\right]. \end{aligned} \tag{3.17}$$

In section 4 we show that for small enough L, X can give a scalar mass contribution to H_d which alters the particle spectrum significantly. The loop contributions to scalar masses discussed above still play a small but significant role while similar contributions to A terms and $B\mu$ are negligible. There are no additional CP-violating phases coming from soft terms since the phases in $M_{1/2}$ and μ can be rotated away and all of the loop contributions are to diagonal (and therefore real) soft masses.[7]

4. Sparticle spectrum and phenomenology

Having introduced the general framework for our model, we now turn to some specific numbers. In order to produce a weak scale μ, we fix l_s (for a given $r_s \sim \mathcal{O}(1)$), such that the scalar VEV of Φ reaches the weak scale in the vicinity of the Higgs fields. Thus, for a given choice of r_s, l_s may be determined such that the resulting μ term induces the correct EWSB radiatively (though in general one must know what the high scale soft masses are in order to know what value of μ that is).

[7]This however does not solve the strong CP problem.

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4.1 Sparticle spectrum

Both scenarios for supersymmetry-breaking have similar "gaugino-dominated" boundary conditions, in which the soft masses for all scalars are smaller than the gaugino masses, with the possible exception of the down-type Higgs, which receives the largest gaugino-loop contributions in scenario 1 as well as sizable supergravity contributions in scenario 2.

This hierarchy in the scalar masses at the GUT scale differs in one very important way with respect to the standard minimal supergravity inspired (SUGRA) models. The fact that we have non-universal scalar masses means that there are contributions to the evolution of the soft masses from the $\mathrm{U}(1)_Y$ D-terms. These D-terms contribute to the beta function of scalar mass m_i^2,

$$\begin{aligned} \frac{d}{dt}\, m_i^2 &= Y_i\, \frac{3}{5} \frac{g_1^2}{16\,\pi^2}\, S\,, \\ S &= \left(m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}\Big[m_Q^2 - 2m_u^2 + m_d^2 - m_L^2 + m_e^2\Big]\right), \end{aligned} \tag{4.1}$$

where g_1 is the $\mathrm{U}(1)_Y$ coupling, normalized appropriately for SU(5) unification, Y_i is (standard) hypercharge for scalar i, and the trace is over the three families. The quantity S is an RG invariant, and thus vanishes at all scales if all scalar masses are equal at some scale. Thus it contributes nothing to the evolution of SUGRA scalar masses. In our model, this term will affect all of the scalar masses, with the effect being the most dramatic for the sleptons, which have large hypercharges and receive no contributions from the strong coupling. This results in a neutralino LSP in a large region of parameter space.

The condition that $B\mu = 0$ at the high scale provides us with a particular moderate to high value of $\tan\beta$ [27] for each choice of $M_{1/2}$ and m_{H_d}. In figure 4 we show the contours of constant $\tan\beta$ in the plane of $M_{1/2}$ and m_{H_d}, including the gaugino-loop contributions given in eq. (4.2). It is remarkable that both the mechanism that generates the 4 d Yukawa couplings of section 2 and the scenarios for supersymmetry breaking favor the same intermediate to large values of $\tan\beta$.

The resulting LSP in the plane of $M_{1/2}$ and m_{H_d} is indicated in figure 4 by the dashed curves.[8] From this figure, we see that if $M_{1/2}$ is much larger than m_{H_d}, we arrive at a $\tilde{e}_R$ LSP because the large $M_{1/2}$ results in a relatively large neutralino mass, while the selectrons get very little contributions from gaugino loops and the D-term contribution to the evolution is small for small m_{H_d}. On the other hand, if m_{H_d} is much larger than $M_{1/2}$, the D-term contribution to the slepton masses

[8]This figure has included scalar masses at the GUT scale introduced in eq. (4.2) below. These contributions to the scalar masses at M_c do not significantly affect the contours of $\tan\beta$, though it does generally alter the LSP curves.

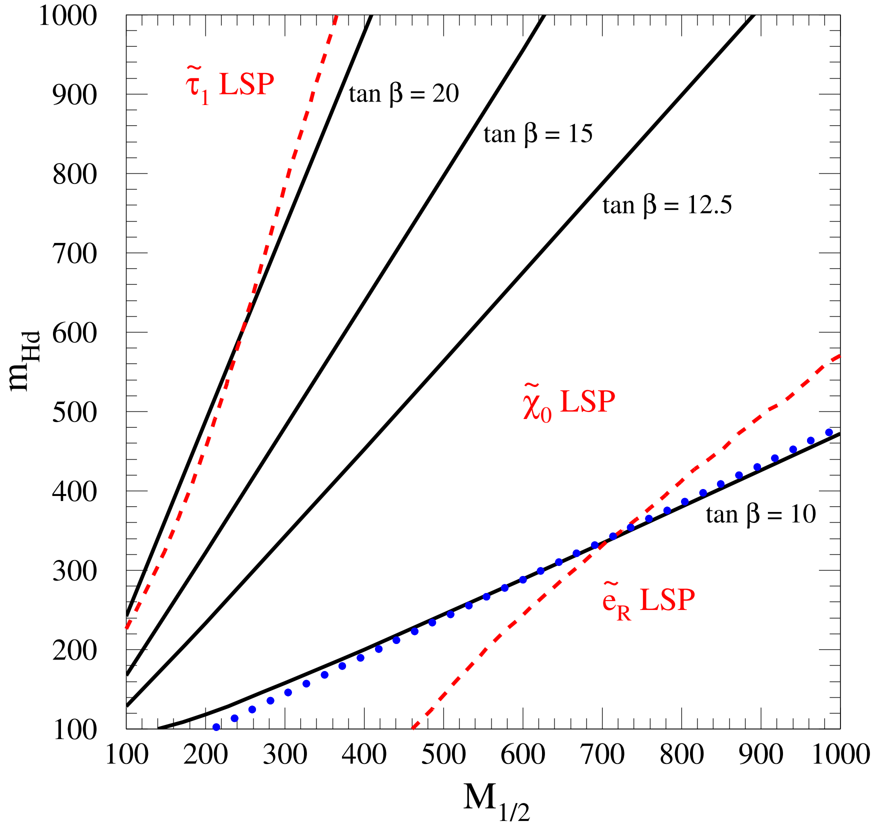

Figure 4: The solid curves are the contours of m_{H_d} for a choice of $M_{1/2}$ (both at the GUT scale) and $\tan\beta$. The dashed curves demark regions where $\widetilde{\tau}_1$, $\widetilde{e}_R$ and $\widetilde{\chi}_1^0$ are the LSP. The dotted curve marks the value of m_{H_d} as a function of $M_{1/2}$ for the example model of scenario 1.

overpowers the standard gaugino contribution. This results in a $\widetilde{\tau}_1$ LSP (which is mostly $\widetilde{\tau}_L$) or a negative m^2 for $\widetilde{\tau}_1$ which spontaneously breaks the electromagnetic symmetry (for consequences of a generic D-term, see [32]). However, for $m_{H_d} \sim M_{1/2}$ the D-term contribution is enough to raise the $\widetilde{e}_R$ mass above the lightest neutralino mass, and does not push the $\widetilde{\tau}_1$ below it, resulting in a stable $\widetilde{\chi}_0$ LSP which provides a suitable dark matter candidate, and results in missing transverse energy signatures at a hadron collider.

4.2 Scenario 1

In this scenario, the gaugino masses arise from a superpotential coupling $X\,\Phi$ which induces a VEV in F_X. X is located a large distance away from the matter fields. Thus, all supergravity contributions to scalar masses are zero because of the small overlap between the distant X field and the matter fields. However, there are contributions from the gaugino-loops in (3.13) which contribute to all of the scalar masses, and thus are all proportional to $M_{1/2}$. Thus, all of the boundary conditions are effectively specified by $M_{1/2}$ and the position of X.

As a particular example, for $|l_x - l_h|/L \sim 0.4$ one obtains,[9]

$$
\begin{aligned}
m^2_{\tilde{f}_1} &= 0.014\, M^2_{1/2}\,, \qquad & m^2_{H_d} &= 0.23\, M^2_{1/2}\,,\\
m^2_{\tilde{f}_2} &= 0.019\, M^2_{1/2}\,, \qquad & m^2_{H_u} &= 0.061\, M^2_{1/2}\,,\\
m^2_{\tilde{f}_3} &= 0.049\, M^2_{1/2}\,. &&
\end{aligned} \tag{4.2}
$$

Thus, our soft terms for squarks are dominated by the gaugino-mediated contributions, and are relatively flavor-blind. The off-diagonal squark matrix entries are then induced by the CKM rotation from interaction to mass eigenbasis and can be estimated for the first two families as,

$$
\frac{\widetilde{m}_d^2 - \widetilde{m}_s^2}{\widetilde{m}_s^2}\, V_{us} \sim 4 \times 10^{-4}\,. \tag{4.3}
$$

This is small enough to avoid the supersymmetry flavor and CP problems generally associated with off-diagonal squark masses. The dotted line in figure 4 shows the value of m_{H_d} for this model. As can be seen, this corresponds to $\tan\beta \sim 10$ and has a neutralino LSP for $M_{1/2} < 700\,\mathrm{GeV}$.

4.3 Scenario 2

In this scenario gaugino masses are determined by the singlet X which develops an auxiliary VEV through superpotential coupling $X\,\Phi^2$. $M_{1/2}$ may be fixed by localizing X a suitable distance between the Higgses and the source for Φ so that $F_X/(M_*^2 L)$ is a weak scale gaugino mass (we assume that $L \sim l_s$, or in other words, that the source for Φ is localized on one end of the compact dimension, and the MSSM matter and Higgs live roughly at the other end). The X field must be localized far enough from the fermions to avoid dangerous supergravity-mediated flavor mixing in the soft masses. We can proceed by choosing r_s and l_s to get a particular weak scale μ and localize X so that an appropriate $M_{1/2}$ results. For some "typical" numbers $r_s = 2$ (and $l_s = -16.5$ so that $\mu \sim 450\,\mathrm{GeV}$), these two constraints require that X be localized in the region $l_x \sim -4.7$ for $r_x \sim 0.5$. This results in a gaugino mass of $M_{1/2} \sim 300$, which is the right order of magnitude to provide the correct EWSB given the value of μ.

Computing the supergravity contributions to the soft masses, we find that all of the squark and the up-type Higgs receive negligible contributions because they are localized too far away from X. On the other hand, the down-type Higgs receives a substantial contribution to its soft mass of about $m^2_{H_d} \sim (250\,\mathrm{GeV})^2$ because it lies relatively close to X. The 5 d loop contributions are also generically small because

[9]Though eq. (3.13) is only an estimate of the loop contributions, it preserves the percentage difference between scalar masses reasonably accurately. Thus, we present figures to two significant digits to better illustrate the mass splittings.

they are loop suppressed, and have been chosen for this example to be the same as those presented in (4.2), and are thus once again safe from the point of view of flavor violation.

By performing a brute force scan through the "reasonable" range of r_s and r_x from 0.5 to 2, we find that this is a generic prediction of the model; we are able to obtain any $M_{1/2}$ and m_{H_d} between 100 GeV and 1 TeV, as well as the corresponding value of μ at the high scale required to induce EWSB from these boundary conditions. The supergravity contributions to other soft masses are always so small as to be negligible, and the gaugino-loop contributions to squark masses are a small factor times $M_{1/2}^2$.

Having determined that our model easily can realize the plane of $M_{1/2}$ and m_{H_d} from the regions of 100 GeV to 1 TeV, and found that we can accommodate the necessary μ for EWSB resulting $\tan\beta$, we can switch our discussion from the underlying model parameters r_s, l_s, r_x and l_x and discuss the resulting theory defined by a particular choice of $M_{1/2}$ and m_{H_d} at scale M_c (which we will take for simplicity to be the GUT scale, $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV). Thus, we may completely specify the supersymmetry breaking parameters of scenario 2 by the two free parameters,

$$M_{1/2}\,, m_{H_d} \tag{4.4}$$

with μ fixed for radiative EWSB and $\tan\beta$ determined by the condition that $B\mu = 0$ at $M_c \sim M_{\rm GUT}$. As shown in figure 4, this results in a particular value of $\tan\beta$ determined by the boundary conditions $B\mu = 0$ at M_c, while μ is determined at the weak scale from the observed mass of the Z boson, and depends very strongly on the choice of $M_{1/2}$ and rather weakly on the choice of m_{H_d}.

4.4 Collider signatures

The resulting weak scale phenomenologies for both supersymmetry breaking scenarios discussed above are similar, and thus we briefly discuss both together here. As in all gaugino-dominated models, the sparticle spectrum is mostly dependent on the choice of $M_{1/2}$. For example, for $M_{1/2} \sim m_{H_d} \sim 200$ (so $\tan\beta \sim 12$), the resulting spectrum has squarks with masses on the order of 430 GeV, charged and neutral sleptons with masses between 80 and 150 GeV, gluinos with mass around 500 GeV, and weak charginos and neutralinos with masses between 75 and 320 GeV, with the lighter states being dominantly gaugino-like. A number of these particles would be accessible at Run II of the Tevatron, with a variety of signals (all characterized by the missing transverse energy from the $\widetilde{\chi}_0$ LSP). Larger values of $M_{1/2}$ will result in a heavier sparticle spectrum, with the qualitative features unchanged; the gluino will tend to be the heaviest superparticle, with the squarks somewhat lighter, followed by sleptons and weak gauginos. Once the superpartner masses become heavier than a few hundred GeV there is insufficient energy to produce them at the Tevatron. However, the CERN Large Hadron Collider (LHC) will probe $M_{1/2}$ up to roughly one TeV. Again, this will be realized through a variety of signals, one particular interesting example of which is the "tri-lepton" signal coming from the leptonic decay of the lighter gauginos into the neutralino LSP [33].

As in all models which can be described at low energies by the MSSM, there are three neutral Higgs bosons (two CP even and one CP odd) and a pair of charged Higgs scalars. The heavy CP even, CP odd, and charged Higgs bosons typically have masses that are a few times larger than $M_{1/2}$ and thus the model exhibits the Higgs "decoupling" limit in which the lightest Higgs boson has approximately SM couplings. For the moderate values of $\tan\beta$ realized by the model, the interesting signals of the pseudo-scalar Higgs produced in association with b quarks may be observed at Tevatron or LHC [34]. The lightest Higgs boson typically has a mass in the range of 110 to 120 GeV, much of which will be probed by LEP [35], with higher masses typically accessible to the Tevatron and/or LHC [36].

5. Conclusions

We have presented a model in which all masses below the GUT scale are generated by localizing fields in a small extra dimension. All dimensionless couplings are of order one, all dimensionful couplings are of order the Planck scale and all distances are of order the Planck length, save the size of the extra dimension which is of order the inverse GUT scale. The quark flavor portion of the model manages to beautifully realize the CKM mixings and CP-violating phase observed in nature, as well the observed quark masses. It is completely independent of the nature of supersymmetry breaking, and thus can be taken as a generic picture of how quark masses might arise from a small extra dimension in a supersymmetric context.

We have explored two different pictures for how the extra dimension might play a role in supersymmetry breaking, both of which are variations of gaugino mediation. While successfully incorporating attractive features such as radiative EWSB and the possibility of a neutral LSP suitable as a dark matter candidate, they result in distinctive boundary conditions for soft masses at the GUT scale, and thus result in interesting relations among superparticle masses not seen in other models. The separation of the families leaves an imprint on the boundary conditions, producing a striking scalar spectrum distinguishable from other predictive models of supersymmetry breaking.

We have employed a bottom-up approach in constructing the model, taking for granted details related to the spontaneous breaking of the GUT symmetry to the SM gauge group, specific details concerning the dynamical localization of the fermions, and the breakdown of the full $N = 1$ supersymmetry in the 5 d theory to the $N = 1$ in the 4 d theory. It would be interesting to pursue these last two technical details with more rigor. Further, it would be interesting to construct a full GUT theory, to explore the possibility that GUT physics stabilizes the size of the extra dimension at

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the GUT scale. Further, the details of the GUT breaking must address the lepton masses observed in nature. In fact, our model contains (at least) two gauge singlet fields which might play the role of the right-handed neutrino.

In general, we find that a small extra dimension allows for a vast array of new possibilities to solve old problems. While extra dimensions of this size would not be accessible at any collider in the near future, their indirect effects on low energy phenomena can be dramatic.

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A. Localizing a chiral superfield in an extra dimension

In this appendix we show how a chiral superfield may be localized in an extra dimension with an exponential profile. Our starting point is the (on-shell) action for an $N = 1$ chiral superfield in 5 space-time dimensions,

$$S = \int d^4x\, dy \left\{ \int d^4\theta \left(\Psi^\dagger \Psi + {\Psi_c}^\dagger \Psi_c \right) + \left(\int d^2\theta\, i\, \Psi_c [\partial_y + M(y)] \Psi + \text{h.c.} \right) \right\}, \tag{A.1}$$

where Ψ and Ψ_c are the left-chiral and charge-conjugated right-chiral $N = 1$ 4 d superfield components of the single $N = 1$ 5 d chiral superfield Ψ. $\partial_y = \partial/\partial y$ denotes the partial derivative with respect to the fifth dimension. This formulation of the action is convenient because it is written in the usual $N = 1$ superfield language and thus is manifestly $N = 1$ supersymmetric. We have written the mass parameter $M(y)$ explicitly as a function of the fifth dimension. This could be realized, for example, by appropriately coupling an additional chiral superfield to $\Psi_c \Psi$ and including dynamics which give its scalar component a VEV.

In terms of component fields, eq. (A.1) may be written,

$$\begin{aligned} S = \int d^4x\, dy \Big\{ & F_L^* F_L + F_R^* F_R + \partial_\mu \phi_L^* \partial^\mu \phi_L + \partial_\mu \phi_R^* \partial^\mu \phi_R + \\ & + i\, \overline{\Psi} \left[\gamma^\mu \partial_\mu - \gamma_5 \partial_y - M(y) \right] \Psi + \\ & + i \Big(F_R^* [\partial_y + M(y)]\, \phi_L + \phi_R^* [\partial_y + M(y)]\, F_L + \\ & \quad + F_L^* [\partial_5 - M(y)]\, \phi_R + \phi_L^* [\partial_y - M(y)]\, F_R \Big) \Big\}, \end{aligned} \tag{A.2}$$

where $\phi_L, P_L\Psi, F_L$ ($\phi_R^*, P_R\Psi, F_R^*$) are the scalar, spinor, and F component fields of Ψ

(Ψ_c), respectively. Expanding these fields in terms of a complete orthonormal basis of scalar functions of y,

$$
\begin{aligned}
P_L\,\Psi(x,y) &= \sum_n \Psi^n_L(x)\, b^n(y)\,, \qquad & P_R\,\Psi(x,y) &= \sum_n \Psi^n_R(x)\, f^n(y)\,,\\
\phi_L(x,y) &= \sum_n \phi^n_L(x)\, B^n(y)\,, \qquad & \phi_R(x,y) &= \sum_n \phi^n_R(x)\, F^n(y)\,,\\
F_L(x,x5) &= \sum_n F^n_L(x)\, \mathcal{B}^n(y)\,, \qquad & F_R(x,x5) &= \sum_n F^n_R(x)\, \mathcal{F}^n(y)\,,
\end{aligned}
\tag{A.3}
$$

we find that the action eq. (A.2) simplifies considerably if one requires,

$$
\begin{aligned}
[\partial_y + M(y)] \left\{ \begin{array}{c} b^n(y) \\ B^n(y) \\ \mathcal{B}^n(y) \end{array} \right\} &= \lambda_n \left\{ \begin{array}{c} f^n(y) \\ \mathcal{F}^n(y) \\ F^n(y) \end{array} \right\},\\
[\partial_y - M(y)] \left\{ \begin{array}{c} f^n(y) \\ F^n(y) \\ \mathcal{F}^n(y) \end{array} \right\} &= -{\lambda_n}^* \left\{ \begin{array}{c} b^n(y) \\ \mathcal{B}^n(y) \\ B^n(y) \end{array} \right\},
\end{aligned}
\tag{A.4}
$$

indicating that $[b^n(y),\, f^n(y)]$, $[B^n(y),\, \mathcal{F}^n(y)]$ and $[\mathcal{B}^n(y),\, F^n(y)]$ are "bosonic" and "fermionic" pairs of solutions to a SUSY Quantum Mechanics problem with $Q = [\partial_y + M(y)]$ [37]. These first order differential equations may be combined to give second order equations for each individual function,

$$
\begin{aligned}
\Big[-\partial_y^2 + M^2(y) - (\partial_y M(y)) \Big] \left\{ \begin{array}{c} b^n(y) \\ B^n(y) \\ \mathcal{B}^n(y) \end{array} \right\} &= |\lambda_n|^2 \left\{ \begin{array}{c} b^n(y) \\ B^n(y) \\ \mathcal{B}^n(y) \end{array} \right\},\\
\Big[-\partial_y^2 + M^2(y) + (\partial_y M(y)) \Big] \left\{ \begin{array}{c} f^n(y) \\ F^n(y) \\ \mathcal{F}^n(y) \end{array} \right\} &= |\lambda_n|^2 \left\{ \begin{array}{c} f^n(y) \\ F^n(y) \\ \mathcal{F}^n(y) \end{array} \right\}.
\end{aligned}
\tag{A.5}
$$

From this result, we see that the same differential equation determines $b^n(y)$, $B^n(y)$ and $\mathcal{B}^n(y)$, and another determines $f^n(y)$, $F^n(y)$ and $\mathcal{F}^n(y)$, and thus there are only two independent sets of functions relevant to the four-dimensional effective theory description (instead of six). This is certainly not surprising; it is an indication that 4 d $N = 1$ supersymmetry remains unbroken. The important point for our purposes is that each component field for a given KK mode of the chiral superfield acquires the same profile in the extra dimension. Putting this expansion into eq. (A.2) and carrying out the dy integration, we arrive at,

$$
\begin{aligned}
S &= \sum_n \int d^4x\, F^{n*}_L\, F^n_L + F^{n*}_R\, F^n_R + \partial_\mu \phi^{n*}_L\, \partial^\mu \phi^n_L + \partial_\mu \phi^{n*}_R\, \partial^\mu \phi^n_R + \\
&\qquad + i\, \overline{\Psi^n}\, [\gamma^\mu\, \partial_\mu + \lambda_n]\, \Psi^n + i\, \lambda_n \Big(F^{n*}_R\, \phi^n_L + \phi^{n*}_R\, F^n_L + F^{n*}_L\, {\phi^n}_R + \phi^{n*}_L\, F^n_R \Big)\\
&= \sum_n \int d^4x \left\{ \int d^4\theta\, \left(\Psi^{n\dagger}\Psi^n + {\Psi_c}^{n\dagger}{\Psi^n}_c \right) + \left(\int d^2\theta\, i\, \lambda_n\, \Psi^n_c\, \Psi^n + \text{h.c.} \right) \right\}
\end{aligned}
\tag{A.6}
$$

which is a free supersymmetric theory for an infinite tower of massive left- plus right-chiral multiplets as well as some number of left- and right-chiral zero mass modes (whose number need not be equal [38]). Introducing interactions to this theory does not change the end result for the kinetic terms, though generically interactions will be induced among all of the various modes, and with varied coupling strengths proportional to the overlap of the profiles of all participating modes in the extra dimension.

We will be considering gaussian profiles for the zero modes, which can be considered "generic" in the sense that given any mass function which crosses zero at some point in y, for a small region about that point, the function may be approximated as linear, $M(y) \approx 2\,\zeta^2 y$, with $2\,\zeta^2$ the slope at the crossing point. An example of such a profile is a domain wall arising from a kink soliton. For this choice of $M(y)$, eq. (A.5) looks like the Schrödinger equation for a harmonic oscillator with frequency $2\,\zeta^2$ and energy shifted by $-\zeta^2$ for the left-chiral and $+\zeta^2$ for the right-chiral modes. Thus, for $\zeta^2 > 0$ there is a single left-chiral zero mode with profile,

$$b^0(y) = \left(\frac{2\,\zeta^2}{\pi}\right)^{(1/4)} e^{i\varphi}\, e^{-\zeta^2 y^2} \tag{A.7}$$

and the higher mode $b^n(y)$ are given by the familiar product of exponentials and Hermite polynomials. The $n \geq 1$ $f^n(y)$ functions may be obtained from eq. (A.4). For $\zeta^2 < 0$ there is a single right-chiral zero mode. Thus, any crossing of zero in $M(y)$ will generally produce a zero mode localized around the zero crossing with gaussian fall-off (the width of the gaussian being determined by the slope of $M(y)$ at the crossing point), provided the deviations from linearity occur sufficiently far away from the crossing point.

Equation (A.7) explicitly includes a phase φ. Such a phase is undetermined by our choice of $M(y)$, and does not contribute to the kinetic terms derived in eq. (A.6). However, it may play a role in the interaction terms for the four-dimensional effective theory, where careful analysis is required to determine which phases are physical, and which may be rotated away by an appropriate redefinition of fields.

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